Eur. Phys. J. C 35, 197–206 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-01817-2

THE EUROPEAN PHYSICAL JOURNAL C

Double-lepton polarization asymmetries in the $B \to K\ell^+\ell^-$ decay beyond the standard model

T.M. Aliev[a], V. Bashiry, M. Savcı[b]

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

Received: 23 November 2003 / Revised version: 8 March 2004 /
Published online: 30 April 2004 –

Abstract. General expressions for the double-lepton polarizations in the $B \to K\ell^+\ell^-$ decay are obtained, using a model independent effective Hamiltonian, including all possible interactions. Correlations between the averaged double-lepton polarization asymmetries and the branching ratio, as well as the averaged single-lepton polarization asymmetry are studied. It is observed that a study of the double-lepton polarization asymmetries can serve as a good test for establishing new physics beyond the standard model.

1 Introduction

Rare B meson decays, induced by flavor-changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ transitions provide a promising testing ground in the search for effects beyond the standard model (SM). The FCNC decays, which are forbidden at tree level in the SM, appear at loop level and are very sensitive to the gauge structure of the SM. Moreover, these decays are also quite sensitive to the present theories beyond the SM. As is well known, $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays are one-loop processes in the SM, governed by the $b \to s\ell^+\ell^-$ transition, at quark level. Because of their loop structures, these decays are suppressed, and the relevant branching ratios in the SM are expected to be of the order of, roughly, 5×10^{-7} for the $B \to K\ell^+\ell^-$ decay, and 1.5×10^{-6} for the $B \to K^*\ell^+\ell^-$ decay, respectively [1–3]. Recently, the Belle [4] and BaBar [5] Collaborations announced the following measurements of the branching ratio for the $B \to K\ell^+\ell^-$ decay:

$$\mathcal{B}(B \to K\ell^+\ell^-) = \begin{cases} \left(4.8^{+1.0}_{-0.9} \pm 0.3 \pm 0.1\right) \times 10^{-7} & [4], \\ \left(0.65^{+0.14}_{-0.13} \pm 0.04\right) \times 10^{-6} & [5]. \end{cases}$$

One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization [6–11]. Polarization of a single-lepton has been studied in the $B \to K^*\ell^+\ell^-$ [6], $B \to X_s\ell^+\ell^-$ [7, 8], $B \to K\ell^+\ell^-$ [9], $B \to \pi(\rho)\ell^+\ell^-$ [10] and $B_s \to \ell^+\ell^-\gamma$ [11] decays. It has been pointed out in [12] that the study of the polarizations of both leptons provides many additional observables which can be measured and would be useful in testing the SM and looking for new physics beyond the SM. Polarization asymmetries and forward–backward asymmetry due to both leptons have been investigated in the $B \to X_s\tau^+\tau^-$ [13], $B \to K^*\tau^+\tau^-$ [14] and $B \to K\tau^+\tau^-$ [15] decays in the minimal supersymmetric model, respectively.

The goal of the present work is to study various double-lepton polarizations in the exclusive $B \to K\ell^+\ell^-$ decay using the most general form of the effective Hamiltonian, including all possible forms of interactions. Moreover, we study the correlation between double-lepton polarizations and single-lepton polarizations. Our purpose in doing so is to find regions in the new Wilson coefficient parameter space in which the branching ratio and single-lepton polarization would agree with the SM prediction while double-lepton polarizations would not. Obviously, if such a region does exist, it is an indication of the fact that the new physics effects can be established by the measurement only of the double-lepton polarizations.

This paper is organized as follows. In Sect. 2, using the most general form of the effective Hamiltonian, we obtain the matrix element of the $B \to K\ell^+\ell^-$ decay in terms of form factors relevant to the $B \to K$ transition and then derive analytical results of double-lepton polarization asymmetries. In Sect. 3, we numerically investigate the correlations of double-lepton asymmetries for the branching ratio. Moreover we analyze the correlation of double-lepton polarization observables to single-lepton polarizations. This section contains also a discussion and our conclusion.

2 Double-lepton polarizations

In this section we calculate the double-lepton polarization asymmetries, using the most general model independent form of the effective Hamiltonian. The effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition in terms of twelve model independent four-Fermi interactions can be written in the

[a] e-mail: taliev@metu.edu.tr
[b] e-mail: savci@metu.edu.tr

following form:

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}\alpha}{\sqrt{2}\pi} V_{ts} V_{tb}^* \times \Big\{ C_{\rm SL}\, \bar{s} {\rm i}\sigma_{\mu\nu} \frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma^\mu \ell + C_{\rm BR}\, \bar{s}{\rm i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma^\mu\ell$$
$$+ C_{\rm LL}^{\rm tot}\, \bar{s}_{\rm L}\gamma_\mu b_{\rm L}\, \bar{\ell}_{\rm L}\gamma^\mu \ell_{\rm L} + C_{\rm LR}^{\rm tot}\, \bar{s}_{\rm L}\gamma_\mu b_{\rm L}\, \bar{\ell}_{\rm R}\gamma^\mu\ell_{\rm R}$$
$$+ C_{\rm RL}\, \bar{s}_{\rm R}\gamma_\mu b_{\rm R}\, \bar{\ell}_{\rm L}\gamma^\mu\ell_{\rm L} + C_{\rm RR}\, \bar{s}_{\rm R}\gamma_\mu b_{\rm R}\, \bar{\ell}_{\rm R}\gamma^\mu \ell_{\rm R}$$
$$+ C_{\rm LRLR}\, \bar{s}_{\rm L} b_{\rm R}\, \bar{\ell}_{\rm L}\ell_{\rm R} + C_{\rm RLLR}\, \bar{s}_{\rm R} b_{\rm L}\, \bar{\ell}_{\rm L}\ell_{\rm R}$$
$$+ C_{\rm LRRL}\, \bar{s}_{\rm L} b_{\rm R}\, \bar{\ell}_{\rm R}\ell_{\rm L} + C_{\rm RLRL}\, \bar{s}_{\rm R} b_{\rm L}\, \bar{\ell}_{\rm R}\ell_{\rm L} + C_{\rm T}\, \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell$$
$$+ {\rm i}C_{\rm TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Big\} \,, \quad (1)$$

where L and R in (1) are

$$L = \frac{1-\gamma_5}{2}\,, \qquad R = \frac{1+\gamma_5}{2}\,,$$

and C_X are the coefficients of the four-Fermi interactions, and $q = p_B - p_K$ is the momentum transfer. Among the twelve Wilson coefficients several already exist in the SM. For example, the coefficients $C_{\rm SL}$ and $C_{\rm BR}$ in the penguin operators correspond to $-2m_s C_7^{\rm eff}$ and $-2m_b C_7^{\rm eff}$ in the SM, respectively. The next four terms in (1) are the vector type interactions with coefficients $C_{\rm LL}^{\rm tot}, C_{\rm LR}^{\rm tot}, C_{\rm RL}$ and $C_{\rm RR}$. Two of these vector interactions containing $C_{\rm LL}^{\rm tot}$ and $C_{\rm LR}^{\rm tot}$ do exist in the SM as well in the form $(C_9^{\rm eff} - C_{10})$ and $(C_9^{\rm eff} + C_{10})$. Therefore we can say that $C_{\rm LL}^{\rm tot}$ and $C_{\rm LR}^{\rm tot}$ describe the sum of the contributions from SM and the new physics and they can be written as

$$C_{\rm LL}^{\rm tot} = C_9^{\rm eff} - C_{10} + C_{\rm LL}\,,$$
$$C_{\rm LR}^{\rm tot} = C_9^{\rm eff} + C_{10} + C_{\rm LR}\,.$$

The terms with coefficients $C_{\rm LRLR}$, $C_{\rm RLLR}$, $C_{\rm LRRL}$ and $C_{\rm RLRL}$ describe the scalar type interactions. The last two terms with the coefficients $C_{\rm T}$ and $C_{\rm TE}$ obviously describe the tensor type interactions.

Exclusive $B \to K\ell^+\ell^-$ decay is described by the matrix element of the effective Hamiltonian over B and K meson states, which can be parametrized in terms of the form factors. It follows from (1) that in order to calculate the amplitude of the $B \to K\ell^+\ell^-$ decay, the following matrix elements are needed:

$$\langle K\,|\bar{s}\gamma_\mu b|\,B\rangle\,,$$
$$\langle K\,|\bar{s}{\rm i}\sigma_{\mu\nu}q^\nu b|\,B\rangle\,,$$
$$\langle K\,|\bar{s}b|\,B\rangle\,,$$
$$\langle K\,|\bar{s}\sigma_{\mu\nu} b|\,B\rangle\,.$$

These matrix elements are defined as follows:

$$\langle K(p_K)\,|\bar{s}\gamma_\mu b|\,B(p_B)\rangle = f_+\left[(p_B+p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2}\, q_\mu\right] + f_0\, \frac{m_B^2 - m_K^2}{q^2}\, q_\mu\,, \quad (2)$$

with $f_+(0) = f_0(0)$,

$$\langle K(p_K)\,|\bar{s}\sigma_{\mu\nu} b|\,B(p_B)\rangle = -{\rm i}\, \frac{f_{\rm T}}{m_B + m_K}\, [(p_B+p_K)_\mu q_\nu - q_\mu (p_B + p_K)_\nu]\,. \quad (3)$$

The matrix elements $\langle K(p_K)\,|\bar{s}{\rm i}\sigma_{\mu\nu}q^\nu b|\,B(p_B)\rangle$ and $\langle K\,|\bar{s}b|\,B\rangle$ can be obtained from (2) and (3). Multiplying both sides of these equations by q^μ and using the equation of motion, we get

$$\langle K(p_K)\,|\bar{s}b|\,B(p_B)\rangle = f_0\, \frac{m_B^2 - m_K^2}{m_b - m_s}\,, \quad (4)$$

$$\langle K(p_K)\,|\bar{s}{\rm i}\sigma_{\mu\nu}q^\nu b|\,B(p_B)\rangle = \frac{f_{\rm T}}{m_B + m_K}\, \left[(p_B+p_K)_\mu q^2 - q_\mu (m_B^2 - m_K^2)\right]\,. \quad (5)$$

Using the definition of the form factors given in (2)– (4), we get the amplitude for the $B \to K\ell^+\ell^-$ decay, which can be written as

$$\mathcal{M}(B \to K\ell^+\ell^-) = \frac{G_{\rm F}\alpha}{4\sqrt{2}\pi} V_{tb}V_{ts}^* \times \left\{\bar{\ell}\gamma^\mu\ell\, [A(p_B+p_K)_\mu + Bq_\mu] \right. \quad (6)$$
$$+ \bar{\ell}\gamma^\mu\gamma_5\ell\, [C(p_B+p_K)_\mu + Dq_\mu] + \bar{\ell}\ell\, Q + \bar{\ell}\gamma_5\ell\, N$$
$$+ 4\bar{\ell}\sigma^{\mu\nu}\ell\, (-iG)\, [(p_B+p_K)_\mu q_\nu - (p_B+p_K)_\nu q_\mu]$$
$$\left. + 4\bar{\ell}\sigma^{\alpha\beta}\ell\, \epsilon_{\mu\nu\alpha\beta}\, H\, [(p_B+p_K)_\mu q_\nu - (p_B+p_K)_\nu q_\mu] \right\}\,.$$

The functions entering (6) are defined as

$$A = (C_{\rm LL}^{\rm tot} + C_{\rm LR}^{\rm tot} + C_{\rm RL} + C_{\rm RR})\, f_+ + 2(C_{\rm BR} + C_{\rm SL})\, \frac{f_{\rm T}}{m_B + m_K}\,,$$
$$B = (C_{\rm LL}^{\rm tot} + C_{\rm LR}^{\rm tot} + C_{\rm RL} + C_{\rm RR})\, f_- - 2(C_{\rm BR} + C_{\rm SL})\, \frac{f_{\rm T}}{(m_B + m_K)q^2}\, (m_B^2 - m_K^2)\,,$$
$$C = (C_{\rm LR}^{\rm tot} + C_{\rm RR} - C_{\rm LL}^{\rm tot} - C_{\rm RL})\, f_+\,,$$
$$D = (C_{\rm LR}^{\rm tot} + C_{\rm RR} - C_{\rm LL}^{\rm tot} - C_{\rm RL})\, f_-\,,$$
$$Q = f_0\, \frac{m_B^2 - m_K^2}{m_b - m_s}\, (C_{\rm LRLR} + C_{\rm RLLR} + C_{\rm LRRL} + C_{\rm RLRL})\,,$$
$$N = f_0\, \frac{m_B^2 - m_K^2}{m_b - m_s}\, (C_{\rm LRLR} + C_{\rm RLLR} - C_{\rm LRRL} - C_{\rm RLRL})\,,$$
$$G = \frac{C_{\rm T}}{m_B + m_K}\, f_{\rm T}\,,$$
$$H = \frac{C_{\rm TE}}{m_B + m_K}\, f_{\rm T}\,, \quad (7)$$

where

$$f_- = (f_0 - f_+)\frac{m_B^2 - m_K^2}{q^2}\,.$$

We see from (6) that the difference from the SM is due to the last four terms only; namely, scalar and tensor type interactions. From the expression of the matrix element given in (6), we get the following result for the dilepton invariant mass spectrum:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(B \to K\ell^+\ell^-) = \frac{G^2\alpha^2 m_B}{2^{14}\pi^5}\left|V_{tb}V_{ts}^*\right|^2 \lambda^{1/2}(1,\hat{r}_K,\hat{s})v\Delta(\hat{s}) , \qquad (8)$$

where $\lambda(1,\hat{r}_K,\hat{s}) = 1 + \hat{r}_K^2 + \hat{s}^2 - 2\hat{r}_K - 2\hat{s} - 2\hat{r}_K\hat{s}$, $\hat{s} = q^2/m_B^2$, $\hat{r}_K = m_K^2/m_B^2$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1-4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity, and $\Delta(\hat{s})$ is

$$\begin{aligned}\Delta = \frac{4m_B^2}{3}\mathrm{Re}\Big[&-96\lambda m_B^3\hat{m}_\ell(AG^*)\\ &+ 24m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(CD^*) + 12m_B\hat{m}_\ell(1-\hat{r}_K)(CN^*)\\ &+ 12m_B^2\hat{m}_\ell^2\hat{s}\,|D|^2 + 3\hat{s}\,|N|^2 + 12m_B\hat{m}_\ell\hat{s}(DN^*)\\ &+ 256\lambda m_B^4\hat{s}v^2\,|H|^2 + \lambda m_B^2(3-v^2)\,|A|^2\\ &+ 3\hat{s}v^2\,|Q|^2 + 64\lambda m_B^4\hat{s}(3-2v^2)\,|G|^2\\ &+ m_B^2\big\{2\lambda - (1-v^2)\big[2\lambda - 3(1-\hat{r}_K)^2\big]\big\}\,|C|^2\Big] . \end{aligned} \qquad (9)$$

We now proceed by calculating the double-polarization asymmetries, i.e., when the polarizations of both leptons are simultaneously measured. We introduce a spin projection operator defined by

$$\Lambda_1 = \frac{1}{2}(1+\gamma_5 \not{s}_i^-) ,$$
$$\Lambda_2 = \frac{1}{2}(1+\gamma_5 \not{s}_i^+) ,$$

for lepton ℓ^- and antilepton ℓ^+, where $i = \mathrm{L,N,T}$ correspond to the longitudinal, normal and transversal polarizations, respectively. Firstly, we define the following orthogonal unit vectors $s^{-\mu}$ in the rest frame of ℓ^- and $s^{+\mu}$ in the rest frame of ℓ^+:

$$\begin{aligned} s_{\mathrm{L}}^{-\mu} &= \left(0,\mathbf{e}_{\mathrm{L}}^-\right) = \left(0,\frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right) ,\\ s_{\mathrm{N}}^{-\mu} &= \left(0,\mathbf{e}_{\mathrm{N}}^-\right) = \left(0,\frac{\mathbf{p}_K\times\mathbf{p}_-}{|\mathbf{p}_K\times\mathbf{p}_-|}\right) ,\\ s_{\mathrm{T}}^{-\mu} &= \left(0,\mathbf{e}_{\mathrm{T}}^-\right) = \left(0,\mathbf{e}_{\mathrm{N}}^-\times\mathbf{e}_{\mathrm{L}}^-\right) ,\\ s_{\mathrm{L}}^{+\mu} &= \left(0,\mathbf{e}_{\mathrm{L}}^+\right) = \left(0,\frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right) ,\\ s_{\mathrm{N}}^{+\mu} &= \left(0,\mathbf{e}_{\mathrm{N}}^+\right) = \left(0,\frac{\mathbf{p}_K\times\mathbf{p}_+}{|\mathbf{p}_K\times\mathbf{p}_+|}\right) ,\\ s_{\mathrm{T}}^{+\mu} &= \left(0,\mathbf{e}_{\mathrm{T}}^+\right) = \left(0,\mathbf{e}_{\mathrm{N}}^+\times\mathbf{e}_{\mathrm{L}}^+\right) , \end{aligned} \qquad (10)$$

where $\mathbf{p}_\mp$ and $\mathbf{p}_K$ are the three-momenta of the leptons $\ell^\mp$ and the K meson in the center of mass frame (CM) of the $\ell^-\,\ell^+$ system, respectively.

The longitudinal unit vectors s_{L}^- and s_{L}^+ are boosted to the CM frame of the $\ell^-\,\ell^+$ system by a Lorentz transformation, giving

$$\begin{aligned} \left(s_{\mathrm{L}}^{-\mu}\right)_{\mathrm{CM}} &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, \frac{E\mathbf{p}_-}{m_\ell|\mathbf{p}_-|}\right) ,\\ \left(s_{\mathrm{L}}^{+\mu}\right)_{\mathrm{CM}} &= \left(\frac{|\mathbf{p}_-|}{m_\ell}, -\frac{E\mathbf{p}_-}{m_\ell|\mathbf{p}_-|}\right) , \end{aligned} \qquad (11)$$

while the vectors $s_{\mathrm{N}}^{\mp\mu}$ and $s_{\mathrm{T}}^{\mp\mu}$ are not changed by the boost.

We can now define the double-lepton polarization asymmetries as in [12]:

$$P_{ij}(\hat{s}) = \frac{\left(\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\mathbf{s}_i^-,\mathbf{s}_j^+) - \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(-\mathbf{s}_i^-,\mathbf{s}_j^+)\right) - \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\mathbf{s}_i^-,-\mathbf{s}_j^+) - \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(-\mathbf{s}_i^-,-\mathbf{s}_j^+)\right)}{\left(\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\mathbf{s}_i^-,\mathbf{s}_j^+) + \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(-\mathbf{s}_i^-,\mathbf{s}_j^+)\right) + \left(\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\mathbf{s}_i^-,-\mathbf{s}_j^+) + \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(-\mathbf{s}_i^-,-\mathbf{s}_j^+)\right)}, \qquad (12)$$

where $i,j = \mathrm{L,\ N,\ T}$, and the first subindex i corresponds to a lepton while the second subindex j corresponds to an antilepton, respectively.

After lengthy calculations we get the following results for the double-polarization asymmetries:

$$\begin{aligned} P_{\mathrm{LL}} = \frac{4m_B^2}{3\Delta} &\times \mathrm{Re}\Big[32\lambda m_B^3\hat{m}_\ell(A^*G) + 24m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(C^*D)\\ &+ 12m_B\hat{m}_\ell(1-\hat{r}_K)(C^*N) + 256\lambda m_B^4\hat{s}v^2\,|H|^2\\ &- 64\lambda m_B^4\hat{s}(1-2v^2)\,|G|^2\\ &- \lambda m_B^2(1+v^2)\,|A|^2 + 12m_B^2\hat{m}_\ell^2\hat{s}\,|D|^2\\ &+ 3\hat{s}\,|N|^2 + 12m_B\hat{m}_\ell\hat{s}(D^*N) + 3\hat{s}v^2\,|Q|^2\\ &- m_B^2\big\{2\lambda - (1-v^2)\big[2\lambda + 3(1-\hat{r}_K)^2\big]\big\}\,|C|^2\Big] , \end{aligned} \qquad (13)$$

$$\begin{aligned} P_{\mathrm{LN}} = \frac{2\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} &\times \mathrm{Im}\big[2m_B\hat{m}_\ell\hat{s}\,\mathrm{Im}(A^*D) + 32m_B^2\hat{m}_\ell^2\hat{s}(D^*G) + \hat{s}(A^*N)\\ &- 16m_B\hat{m}_\ell\hat{s}(G^*N) - \hat{s}v^2(C^*Q)\\ &+ 2m_B\hat{m}_\ell(1-\hat{r}_K)(A^*C) + 32m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(C^*G)\big] , \end{aligned} \qquad (14)$$

$$\begin{aligned} P_{\mathrm{NL}} = \frac{2\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} &\times \mathrm{Im}\big[-2m_B\hat{m}_\ell\hat{s}(A^*D) - 32m_B^2\hat{m}_\ell^2\hat{s}(D^*G) - \hat{s}(A^*N)\\ &+ 16m_B\hat{m}_\ell\hat{s}(G^*N) - \hat{s}v^2(C^*Q)\\ &- 2m_B\hat{m}_\ell(1-\hat{r}_K)(A^*C) - 32m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(C^*G)\big] , \end{aligned} \qquad (15)$$

$$\begin{aligned} P_{\mathrm{LT}} = \frac{2\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} &\times \mathrm{Re}\Big[2m_B\hat{m}_\ell(1-\hat{r}_K)v\,|C|^2 + 2m_B\hat{m}_\ell\hat{s}v(C^*D)\\ &+ \hat{s}v(C^*N) - \hat{s}v(A^*Q) + 16m_B\hat{m}_\ell\hat{s}v(G^*Q)\big] , \end{aligned} \qquad (16)$$

$$P_{\rm TL} = \frac{2\pi m_B^3\sqrt{\lambda\hat{s}}}{\hat{s}\Delta} \times {\rm Re}\Big[2m_B\hat{m}_\ell(1-\hat{r}_K)v\,|C|^2 + 2m_B\hat{m}_\ell\hat{s}v(C^*D) + \hat{s}v(C^*N) + \hat{s}v(A^*Q) - 16m_B\hat{m}_\ell\hat{s}v(G^*Q)\Big]\ , \quad (17)$$

$$P_{\rm NT} = \frac{8m_B^2 v}{3\Delta} \times {\rm Im}\big[-32\lambda m_B^3\hat{m}_\ell(A^*H) + 128\lambda m_B^4\hat{s}(G^*H) + 6m_B\hat{m}_\ell\hat{s}(D^*Q) + 3\hat{s}(N^*Q) - 2\lambda m_B^2(A^*C) - 32\lambda m_B^3\hat{m}_\ell(C^*G) + 6m_B\hat{m}_\ell(1-\hat{r}_K)(C^*Q)\big]\ , \quad (18)$$

$$P_{\rm TN} = \frac{8m_B^2 v}{3\Delta} \times {\rm Im}\big[-32\lambda m_B^3\hat{m}_\ell(A^*H) + 128\lambda m_B^4\hat{s}(G^*H) + 6m_B\hat{m}_\ell\hat{s}(D^*Q) + 3\hat{s}(N^*Q) + 2\lambda m_B^2(A^*C) + 32\lambda m_B^3\hat{m}_\ell(C^*G) + 6m_B\hat{m}_\ell(1-\hat{r}_K)(C^*Q)\big]\ , \quad (19)$$

$$P_{\rm TT} = \frac{4m_B^2}{3\Delta} \times {\rm Re}\Big[32\lambda m_B^3\hat{m}_\ell(A^*G) - 24m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(C^*D) - 12m_B\hat{m}_\ell(1-\hat{r}_K)(C^*N) - 256\lambda m_B^4\hat{s}v^2\,|H|^2 - 64\lambda m_B^4\hat{s}(1-2v^2)\,|G|^2 - \lambda m_B^2(1+v^2)\,|A|^2 - 12m_B^2\hat{m}_\ell^2\hat{s}\,|D|^2 - 3\hat{s}\,|N|^2 - 12m_B\hat{m}_\ell\hat{s}(D^*N) + 3\hat{s}v^2\,|Q|^2 + m_B^2\big\{2\lambda - (1-v^2)\big[2\lambda + 3(1-\hat{r}_K)^2\big]\big\}\,|C|^2\Big]\ , \quad (20)$$

$$P_{\rm NN} = \frac{4m_B^2}{3\Delta} \times {\rm Re}\Big[96\lambda m_B^3\hat{m}_\ell(A^*G) + 256\lambda m_B^4\hat{s}v^2\,|H|^2 - 3\hat{s}v^2\,|Q|^2 + 12m_B^2\hat{m}_\ell^2\hat{s}\,|D|^2 + 3\hat{s}\,|N|^2 + 12m_B\hat{m}_\ell\hat{s}(D^*N) - \lambda m_B^2(3-v^2)\,|A|^2 - 64\lambda m_B^4\hat{s}(3-2v^2)\,|G|^2 + m_B^2\big\{2\lambda - (1-v^2)\big[2\lambda - 3(1-\hat{r}_K)^2\big]\big\}\,|C|^2 + 24m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(C^*D) + 12m_B\hat{m}_\ell(1-\hat{r}_K)(C^*N)\Big]\ . \quad (21)$$

3 Numerical results and discussion

In this section we present the numerical analysis of all possible double-lepton polarizations, whose explicit expressions we have given in the previous section.

The values of the input parameters used in this work are $|V_{tb}V_{ts}^*| = 0.0385$, $(C_9^{\rm eff})^{sh} = 4.344$, $C_{10} = -4.669$, $\Gamma_B = 4.22\times 10^{-13}$ GeV. It is well known that the Wilson coefficient $C_9^{\rm eff}$ receives long distance contribution coming from the real intermediate J/ψ family. However, in the present work we consider only the short distance contribution. The modulo of $C_7^{\rm eff}$ is fixed by the experimental value of $\mathcal{B}(B \to X_s\gamma)$, while its sign is determined by the SM. In our further analysis we use $(C_7^{\rm eff})_{\rm SM} = -0.313$, and for the parametrization of the form factors we use the results of the first reference in [3].

The region for the new Wilson coefficients can be obtained from the existing experimental results of the BaBar and BELLE Collaboration on $\mathcal{B}(B \to K\ell^-\ell^+)$ [4,5] (see the figures below).

It follows from (13)– (21) that the double-lepton polarization asymmetries depend on q^2 and the new Wilson coefficients. Therefore, it may experimentally be difficult to study these dependences at the same time. For this reason, we eliminate the q^2 dependence by performing the integration over q^2 in the allowed region, i.e., we consider the averaged double-lepton polarization asymmetries. The averaging over q^2 is defined as

$$\langle P_{ij}\rangle = \frac{\displaystyle\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} P_{ij}\frac{{\rm d}\mathcal{B}}{{\rm d}\hat{s}}{\rm d}\hat{s}}{\displaystyle\int_{4\hat{m}_\ell^2}^{(1-\sqrt{\hat{r}_K})^2} \frac{{\rm d}\mathcal{B}}{{\rm d}\hat{s}}{\rm d}\hat{s}}\ .$$

We present our analysis in a series of figures. In Figs. 1–4, we depict the correlation of the averaged double-lepton asymmetries for the branching ratio for the $B \to K\mu^-\mu^+$ decay. Note that the region of the branching ratio is taken from the existing experimental result, and the corresponding regions of variation of the new Wilson coefficients are given in the figures.

From these figures we deduce the following results.
(1) There exist regions of new Wilson coefficients where $\langle P_{\rm LL}\rangle$ departs from the SM result considerably when $\mathcal{B}(B \to K\mu^-\mu^+)$ is very close to the SM value.
(2) $\langle P_{\rm LN}\rangle$ as well as $\langle P_{\rm NL}\rangle$ seem to exceed the SM value 3–4 times, and they change their signs when new Wilson coefficients vary in the allowed region and branching ratio is very close to the SM result. This behavior can serve as a good test for establishing new physics beyond the SM.
(3) In the presence of the new Wilson coefficients, the value of $\langle P_{\rm LT}\rangle$ ($\langle P_{\rm TL}\rangle$) is 3–4 times smaller (larger) compared to the SM prediction. Moreover, $\langle P_{\rm TL}\rangle$ changes its sign when new Wilson coefficients vary.

We do not present the correlation of $\langle P_{\rm NN}\rangle$, $\langle P_{\rm NT}\rangle$, $\langle P_{\rm TN}\rangle$ and $\langle P_{\rm TT}\rangle$ on the branching ratio, since the values of $\langle P_{\rm NN}\rangle$, $\langle P_{\rm NT}\rangle$ and $\langle P_{\rm TN}\rangle$ are very small, and the behavior of $\langle P_{\rm TT}\rangle$ is quite similar to that of $\langle P_{\rm TL}\rangle$. A change in the values of $\langle P_{\rm NT}\rangle$ and $\langle P_{\rm TN}\rangle$ is observed, but no change in their signs seems to occur.

In Figs. 5–13, we present the correlation of $\langle P_{ij}\rangle$ on branching ratio for the $B \to K\tau^+\tau^-$ decay. Similar to the $B \to K\mu^+\mu^-$ decay, one concludes that several $\langle P_{ij}\rangle$ are sizable and sensitive to the existence of new physics. It should be noted that in the present analysis we change the branching ratio in the region $(1 \div 3.5)\times 10^{-7}$.

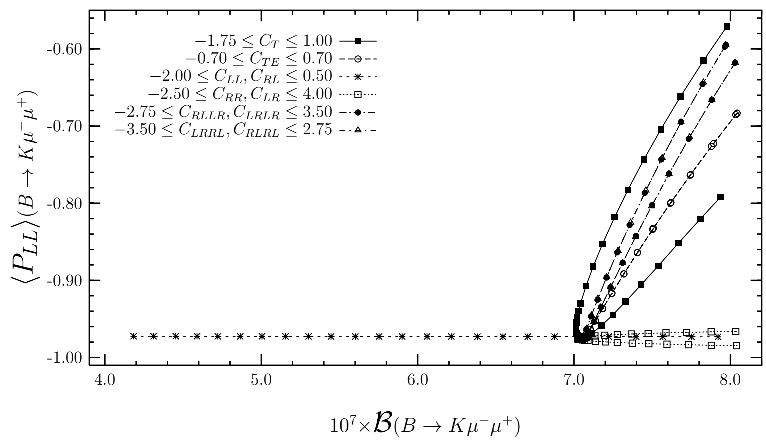

Fig. 1. Parametric plot of the correlation between the averaged double-lepton polarization asymmetry $\langle P_{\rm LL} \rangle$ and the branching ratio for the $B \to K\mu^+\mu^-$ decay, when both leptons are longitudinally polarized

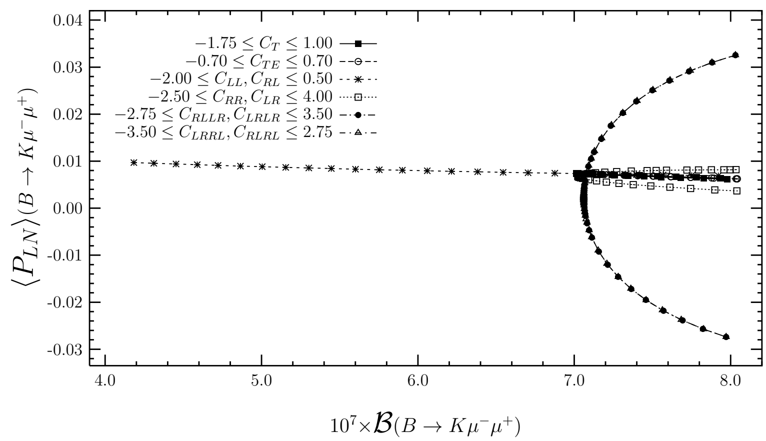

Fig. 2. The same as in Fig. 1, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm LN} \rangle$, when one of the leptons is longitudinally, and the other is normally polarized

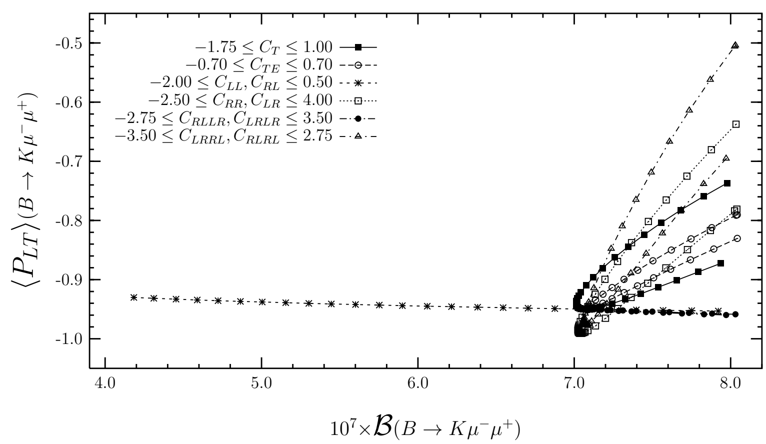

Fig. 3. The same as in Fig. 2, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm LT} \rangle$

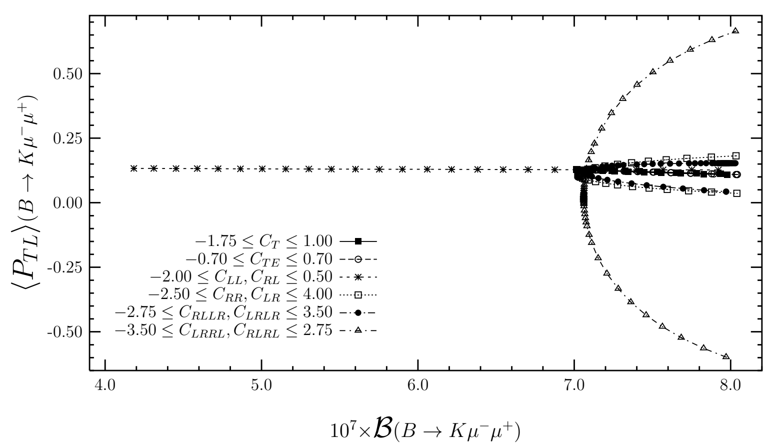

Fig. 4. The same as in Fig. 2, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm TL} \rangle$

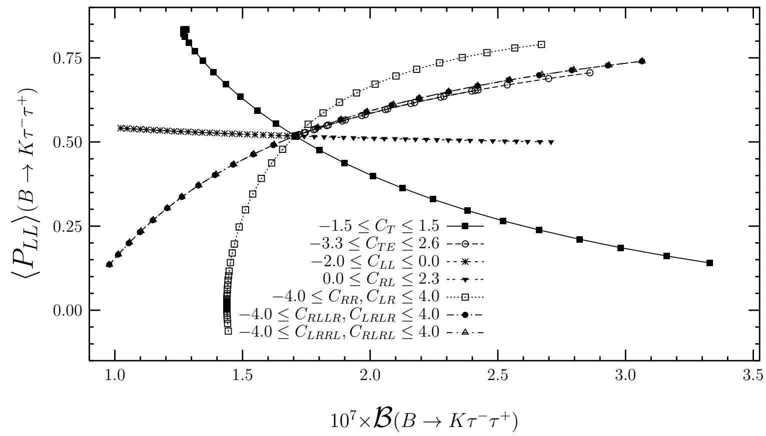

Fig. 5. The same as in Fig. 1, but for the $B \to K\tau^+\tau^-$ decay

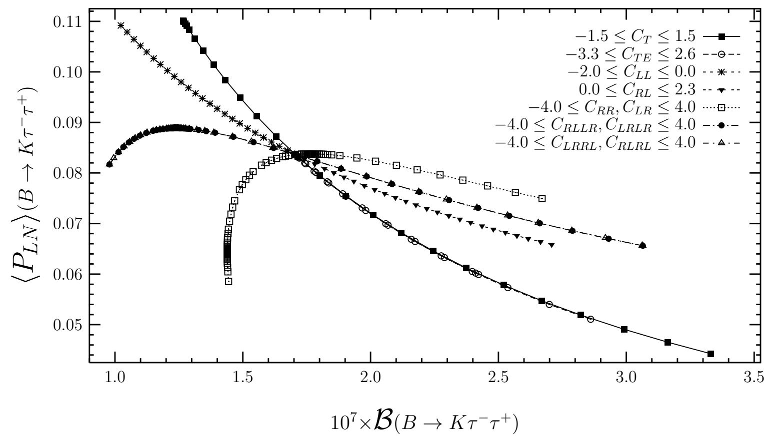

Fig. 6. The same as in Fig. 2, but for the $B \to K\tau^+\tau^-$ decay

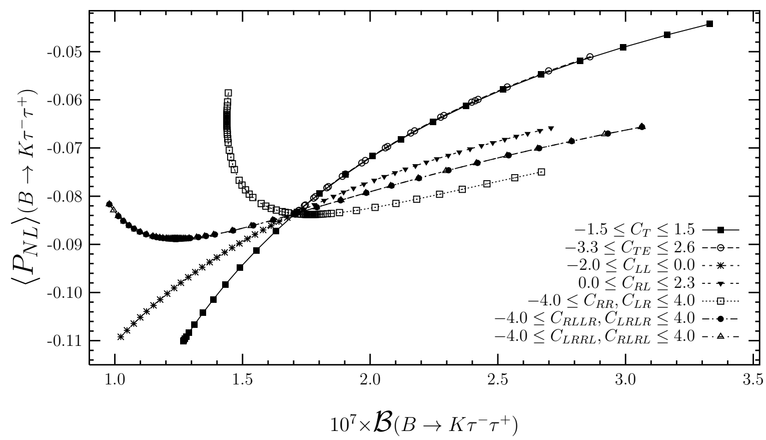

Fig. 7. The same as in Fig. 5, but for the averaged double-lepton polarization asymmetry $\langle P_{\mathrm{NL}} \rangle$

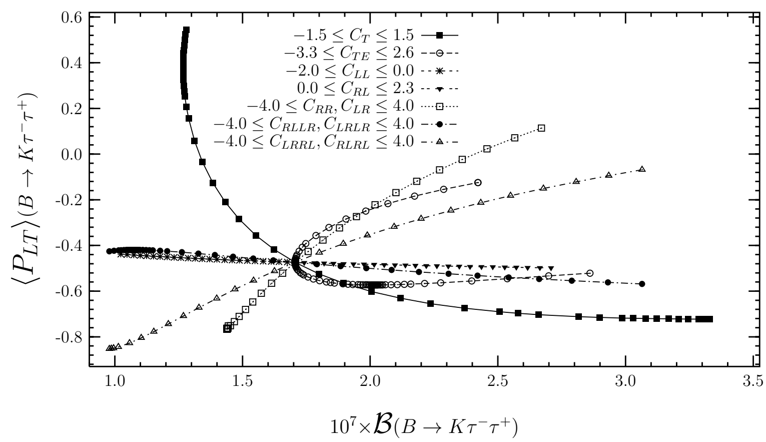

Fig. 8. The same as in Fig. 3, but for the $B \to K\tau^+\tau^-$ decay

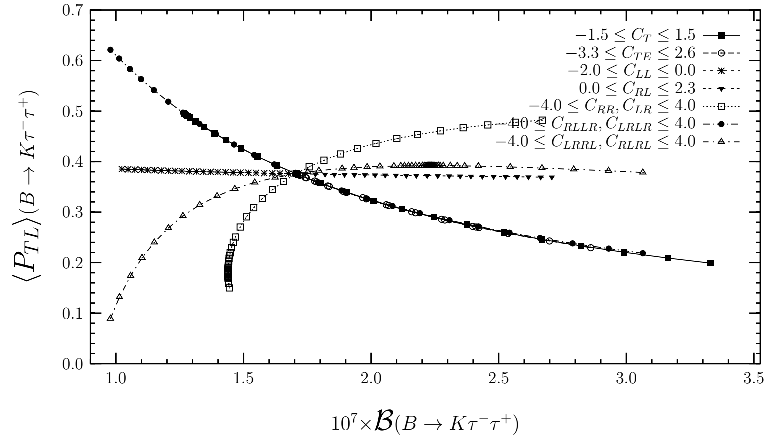

Fig. 9. The same as in Fig. 4, but for the $B \to K\tau^+\tau^-$ decay

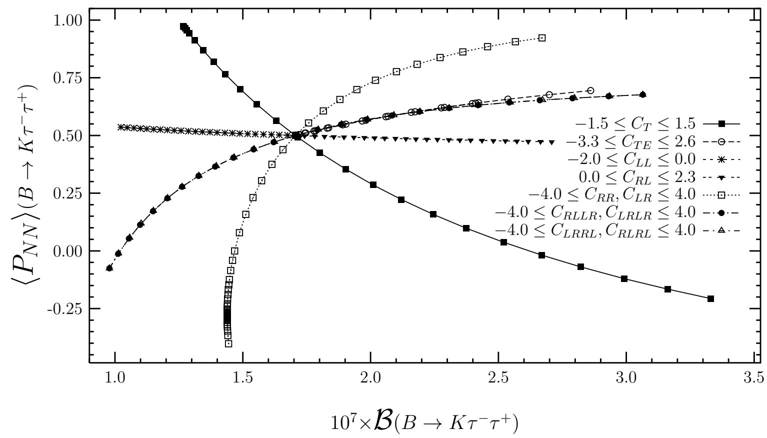

Fig. 10. The same as in Fig. 5, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm NN} \rangle$

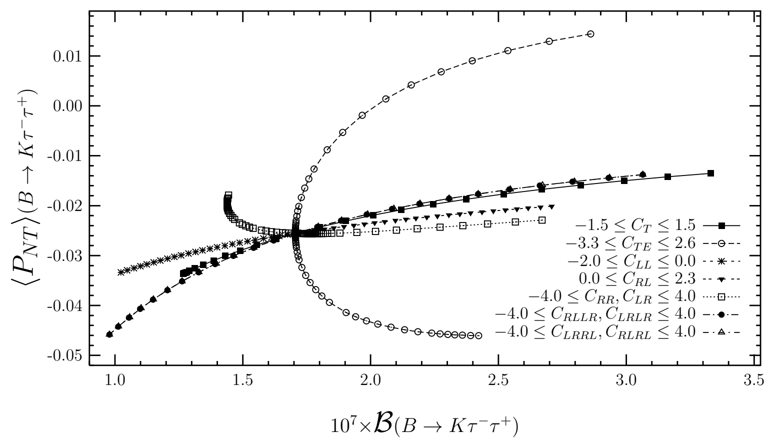

Fig. 11. The same as in Fig. 5, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm NT} \rangle$

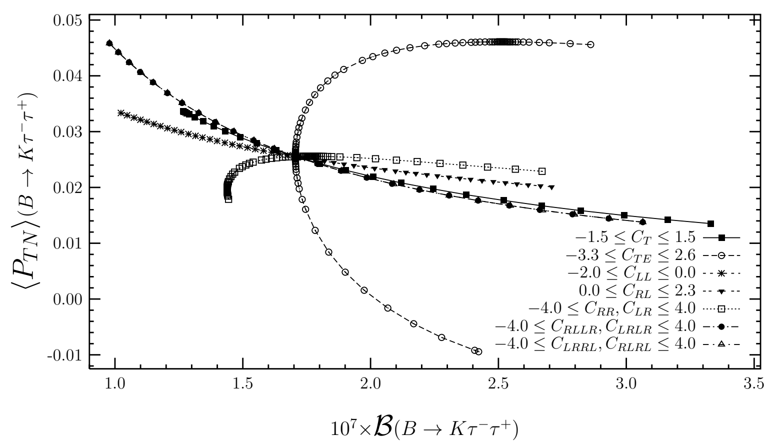

Fig. 12. The same as in Fig. 5, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm TN} \rangle$

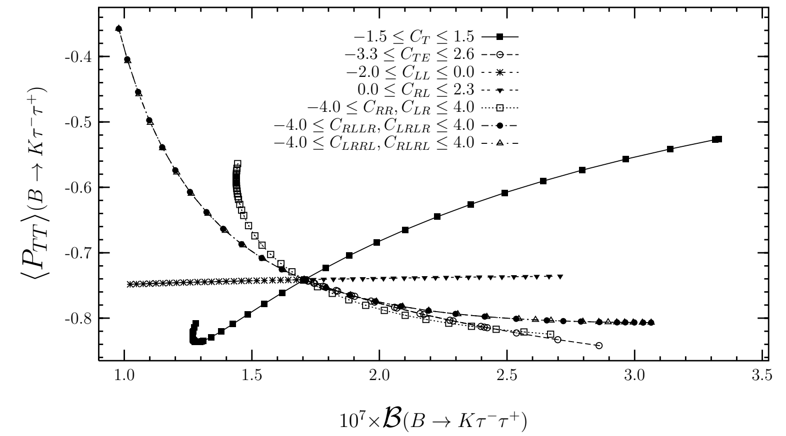

Fig. 13. The same as in Fig. 5, but for the averaged double-lepton polarization asymmetry $\langle P_{\rm TT} \rangle$, when both leptons are transversally polarized

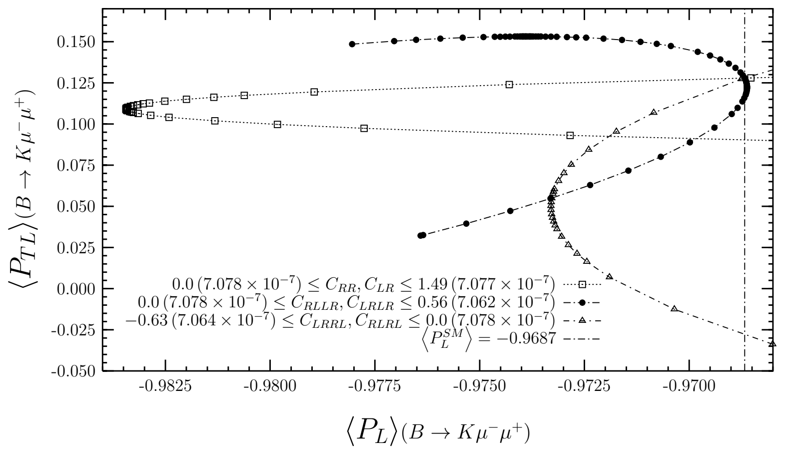

Fig. 14. Parametric plot of the correlation between the averaged double-lepton polarization asymmetry $\langle P_{\rm TL} \rangle$ and the single-lepton polarization $\langle P_{\rm L} \rangle$ for the $B \to K\mu^+\mu^-$ decay. The numbers in parentheses are the values of the branching ratio corresponding to the respective lower and upper values of the new Wilson coefficients

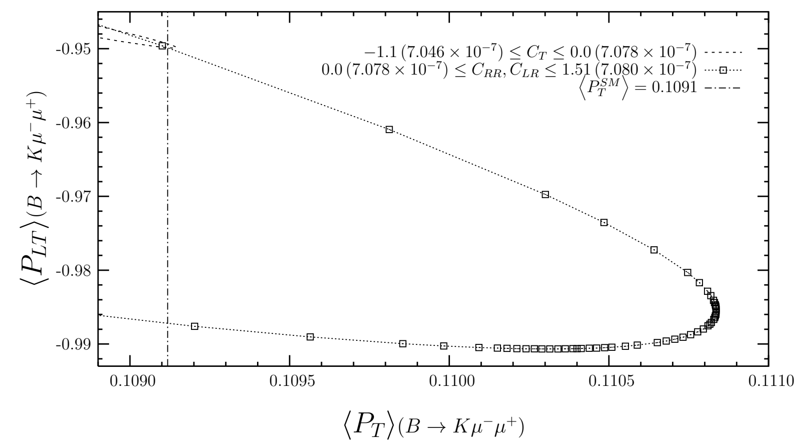

Fig. 15. The same as in Fig. 14, but for the correlation between $\langle P_{\rm LT} \rangle$ and $\langle P_{\rm T} \rangle$ pair

Next, we want to discuss the following problem. Can we establish the new physics effects only by measuring the double-lepton polarization? In other words, do sizable regions of new Wilson coefficients exist, for which the single-lepton polarization coincides with the SM result while double-lepton polarizations do not? In order to analyze this possibility, we study the correlations of averaged double- $\langle P_{ij} \rangle$ and single-lepton $\langle P_i \rangle$ polarizations. We vary the new Wilson coefficients in the region allowed by the measured branching ratio.

Our numerical analysis shows that, for the $B \to K\mu^+\mu^-$ case, the correlations $(\langle P_{\rm TL} \rangle,\ \langle P_{\rm L} \rangle)$ and $(\langle P_{\rm LT} \rangle,\ \langle P_{\rm T} \rangle)$ are more informative. The correlations $(\langle P_{\rm LL} \rangle,\ \langle P_{\rm L} \rangle)$ and $(\langle P_{\rm TT} \rangle,\ \langle P_{\rm T} \rangle)$ are not suitable since their values in the SM are practically the same, and if the new Wilson coefficients are taken into account in the allowed region, the departure of $\langle P_{\rm LL} \rangle$ and $\langle P_{\rm TT} \rangle$ from their SM values is very small. In Figs. 14 and 15 we present the correlations of $\langle P_{\rm TL} \rangle$ with $\langle P_{\rm L} \rangle$ and $\langle P_{\rm LT} \rangle$ on $\langle P_{\rm T} \rangle$, respectively. From these figures we observe that there exist regions of the new Wilson coefficients where double-lepton polarizations differ from the SM while single-lepton polarizations coincide with the SM prediction. Here in this figure and in the rest of the following ones, the numbers in parentheses are the values of the branching ratio corresponding to the respective lower and upper values of the new Wilson coefficients.

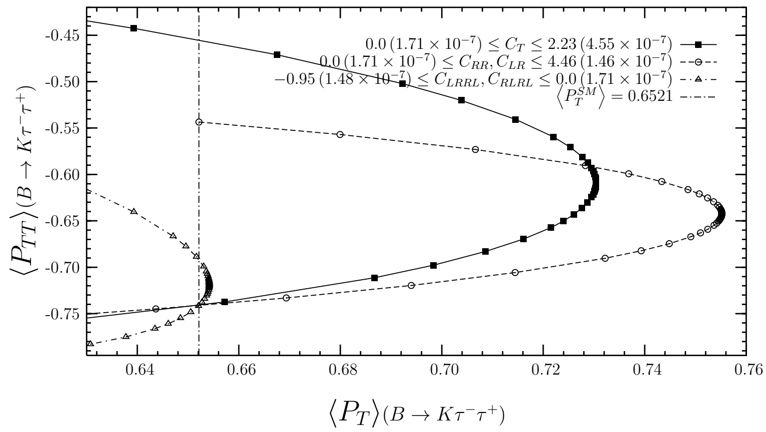

Fig. 16. The same as in Fig. 14, but for the correlation between $\langle P_{\rm TT}\rangle$ and $\langle P_{\rm T}\rangle$ pair, for the $B \to K\tau^+\tau^-$ decay

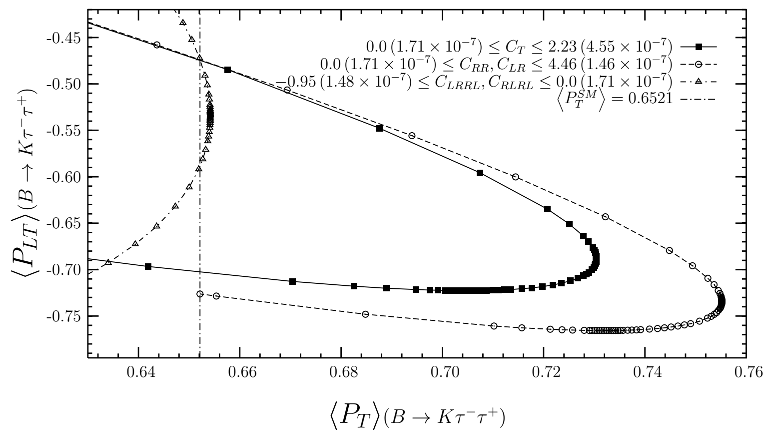

Fig. 17. The same as in Fig. 16, but for the correlation between $\langle P_{\rm LT}\rangle$ and $\langle P_{\rm T}\rangle$ pair

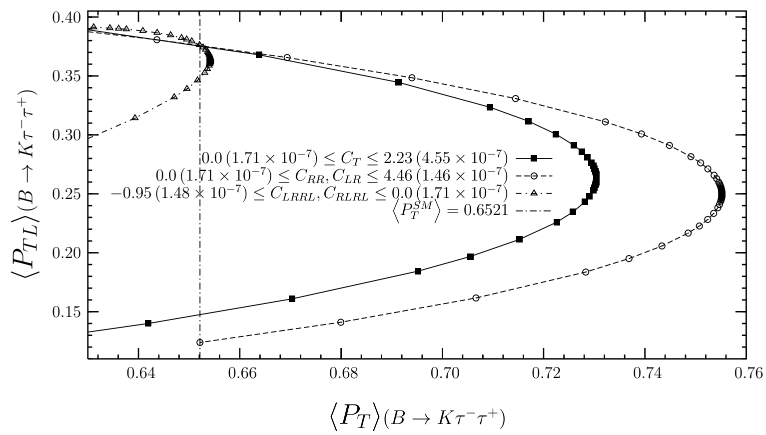

Fig. 18. The same as in Fig. 16, but for the correlation between $\langle P_{\rm TL}\rangle$ and $\langle P_{\rm T}\rangle$ pair

The situation for the $B \to K\tau^+\tau^-$ decay is slightly different. We obtain the result that the study of all correlations between double- and single-lepton polarizations leads to a strong restriction on the tensor type Wilson coefficient $C_{\rm T}$. Besides, analyses of the correlations $(\langle P_{\rm TT}\rangle,\ \langle P_{\rm T}\rangle)$, $(\langle P_{\rm LT}\rangle,\ \langle P_{\rm T}\rangle)$ and $(\langle P_{\rm TL}\rangle,\ \langle P_{\rm T}\rangle)$ show that there exist regions of the new Wilson coefficients $C_{\rm RR}$, $C_{\rm LR}$ and scalar type coefficients $C_{\rm LRRL}$, $C_{\rm RLRL}$ where double-lepton polarizations differ from the SM results, but single-lepton polarizations coincide with that of the SM (see Figs. 16, 17, and 18).

Finally, let us briefly discuss the problem of the detectability of the lepton polarization asymmetries in experiments. Experimentally, to measure an asymmetry $\langle P_{ij}\rangle$ of the decay with the branching ratio $\mathcal{B}$ at $n\sigma$ level, the required relevant number of events (i.e., the number of $B\bar{B}$ pairs) are given by the expression

$$N = \frac{n^2}{\mathcal{B} s_1 s_2 \langle P_{ij}\rangle^2} \ ,$$

where s_1 and s_2 are the efficiencies of the leptons. Typical values of the efficiencies of the τ-leptons range from 50% to 90% for their various decay modes (see for example [16] and references therein). It should be noted here that the error in the τ-lepton polarization is estimated to be about

$(10 \div 15)\%$ [17]. So, the error in the measurement of the τ-lepton asymmetries is of the order of $(20 \div 30)\%$, and the error in obtaining the number of events is about 50%.

It follows from the expression for N that, in order to observe the lepton polarization asymmetries in $B \to K\mu^+\mu^-$ and $B \to K\tau^+\tau^-$ decays at 3σ level, the minimum number of required events are (for the efficiency of the τ-lepton we take 0.5)

(1) for the $B \to K\mu^+\mu^-$ decay

$$N = \begin{cases} 3.5 \times 10^7 & (\text{for } \langle P_{\rm LL}\rangle, \langle P_{\rm LT}\rangle)\ , \\ 5.0 \times 10^8 & (\text{for } \langle P_{\rm TL}\rangle)\ , \\ 2.0 \times 10^{11} & (\text{for } \langle P_{\rm LN}\rangle)\ ; \end{cases}$$

(2) for the $B \to K\tau^+\tau^-$ decay

$$N = \begin{cases} (1.0 \pm 0.5) \times 10^9 & (\text{for } \langle P_{\rm LL}\rangle, \langle P_{\rm LT}\rangle, \langle P_{\rm TL}\rangle, \langle P_{\rm NN}\rangle)\ , \\ (5.0 \pm 2.5) \times 10^8 & (\text{for } \langle P_{\rm TT}\rangle)\ , \\ (4.0 \pm 2.0) \times 10^{10} & (\text{for } \langle P_{\rm LN}\rangle, \langle P_{\rm NL}\rangle)\ , \\ (3.0 \pm 1.5) \times 10^{11} & (\text{for } \langle P_{\rm NT}\rangle, \langle P_{\rm TN}\rangle)\ . \end{cases}$$

On the other hand, the number of $B\bar{B}$ pairs that are produced at B-factories and LHC are about $\sim 5 \times 10^8$ and 10^{12}, respectively. As a result of the comparison of these numbers and N, we conclude that, except for $\langle P_{\rm LN}\rangle$ in the $B \to K\mu^+\mu^-$ decay and $\langle P_{\rm NT}\rangle$, $\langle P_{\rm TN}\rangle$ in the $B \to K\tau^+\tau^-$ decay, all double lepton polarizations can definitely be detectable at LHC. The numbers for the $B \to K\mu^+\mu^-$ decay presented above demonstrate that $\langle P_{\rm LL}\rangle$ and $\langle P_{\rm LT}\rangle$ for the $B \to K\mu^+\mu^-$ decay should be accessible at B-factories after several years of running.

In summary, in this work we present the most general analysis of the double-lepton polarization asymmetries in the $B \to K\ell^+\ell^-$ decay using the most general, model independent form of the effective Hamiltonian. In our analysis we have used the experimental result of the branching ratio for the $B \to K\mu^+\mu^-$ decay announced by the BaBar and BELLE Collaborations. The correlation of the averaged double-lepton polarization asymmetries on the branching ratio (we use the experimental result for the varying region of the branching ratio for the $B \to K\mu^+\mu^-$ decay). We find that the study of double-lepton polarization asymmetries can serve as a good test for establishing new physics beyond the SM. Moreover, we study the correlations between double- and single-lepton polarization asymmetries and observe that there exist regions of the new Wilson coefficients for which double-lepton polarization asymmetries depart considerably from the SM while single-lepton polarization coincides with that of the SM predictions. In other words, in these regions of the new Wilson coefficients only double-lepton polarization asymmetry measurements can establish new physics beyond the SM.

References

1. G. Burdman, Phys. Rev. D **52**, 6400 (1995); J.L. Hewett, Phys. Rev. D **55**, 5549 (1997)
2. A. Ali et al., Phys. Rev. D **66**, 034002 (2002); A. Ali et al., Phys. Rev. D **61**, 074024 (2000); P. Colangelo et al., Phys. Rev. D **53**, 3672 (1996); D. Melikhov, N. Nikitin, Simula, Phys. Rev. D **57**, 6814 (1998)
3. T.M. Aliev, H. Koru, A. Özpineci, M. Savcı, Phys. Lett. B **400**, 194 (1997); T.M. Aliev, A. Özpineci, M. Savcı, Phys. Rev. D **56**, 4200 (1997); C.H. Chen, C.Q. Geng, Phys. Rev. D **66**, 094018 (2002)
4. A. Ishikawa, BELLE Collaboration, Phys. Rev. Lett. **91**, 261601 (2003)
5. B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. **91**, 221802 (2003)
6. T.M. Aliev, M.K. Çakmak, M. Savcı, Nucl. Phys. B **607**, 305 (2001); T.M. Aliev, M. Savci, Phys. Lett. B **481**, 275 (2000)
7. S. Rai Choudhury, A. Gupta, N. Gaur, Phys. Rev. D **60**, 115004 (1999); S. Fukae, C.S. Kim, T. Yoshikawa, Phys. Rev. D **61**, 074015 (2000); T.M. Aliev, M.K. Çakmak, A. Özpineci, M. Savcı, Phys. Rev. D **64**, 055007 (2001); D. Guetta, E. Nardi, Phys. Rev. D **58**, 012001 (1998)
8. F. Krüger, L.M. Sehgal, Phys. Lett. B **380**, 199 (1996); J.L. Hewett, Phys. Rev. D **53**, 4964 (1996)
9. T.M. Aliev, M.K. Çakmak, A. Özpineci, M. Savcı, Phys. Rev. D **64**, 055007 (2001); T.M. Aliev, M. Savcı, A. Özpineci, H. Koru, J. Phys. G **24**, 49 (1998)
10. S.R. Choudhury, N. Gaur, Phys. Rev. D **66**, 094015 (2002)
11. S. Rai Choudhury, N. Gaur, N. Mahajan, Phys. Rev. D **66**, 054003 (2002); E.O. İltan, G. Turan, Phys. Rev. D **61**, 034010 (2000); T.M. Aliev, A. Özpineci, M. Savcı, Phys. Lett. B **520**, 69 (2001)
12. W. Bensalem, D. London, N. Sinha, R. Sinha, Phys. Rev. D **67**, 034007 (2003)
13. N. Gaur, hep-ph/0305242; A.S. Cornell, N. Gaur, JHEP **0309**, 030 (2003)
14. S.R. Choudhury, N. Gaur, A.S. Cornell, G.C. Joshi, Phys. Rev. D **68**, 054016 (2003)
15. S.R. Choudhury, A.S. Cornell, N. Gaur, G.C. Joshi, hep-ph/0307276
16. G. Abbiendi et al., OPAL Collaboration, Phys. Lett. B **492**, 23 (2000)
17. A. Rouge, Workshop on τ lepton physics, Orsay, France (1990)